Formas Quadráticas e Autovalor/Autoveto

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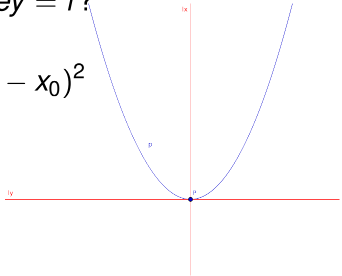
Gods doesn't worry about our mathemathical difficulties
He integrates empirically
Albert Einstein

Curvas Quadráticas

- Qq é: $ax^2 + by^2 + 2cxy + dx + ey = f$?
- Parábola:

$$y - y_0 = a(x - x_0)^2$$



$$y = Ax^2 + Bx + C = A\left[\left(x + \frac{B}{2A}\right)^2 - \frac{B^2}{4A^2} + \frac{C}{A}\right] \Leftrightarrow$$

$$y + \frac{\Delta}{4A} = A(x - \frac{B}{2A})^2 \Rightarrow (x_0, y_0) = \left(-\frac{B}{2A}, -\frac{\Delta}{4A}\right)$$

$$y_0 = Ax_0^2 + C$$

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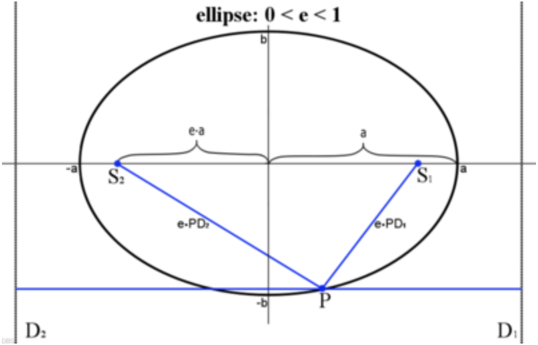
Elipse

- $$\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2} = 1$$



Hiperbole

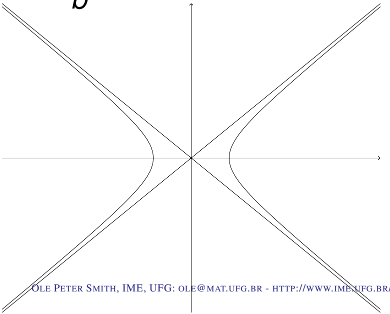
- $$\frac{(x-x_0)^2}{a^2} - \frac{(y-y_0)^2}{b^2} = 1$$

  Ou:

  $$-\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1$$

  Assintodas:

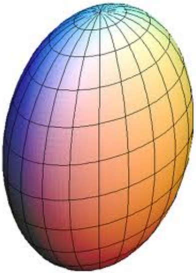
  $$\frac{(x-x_0)^2}{a^2} - \frac{(y-y_0)^2}{b^2} = 0 \Leftrightarrow y - y_0 = \pm\frac{a}{b}(x-x_0)$$



Superfíces Quadráticas

- Qq é:
  $ax^2 + by^2 + cz^2 + 2dxy + 2exz + 2fyz + gx + hy + iz = j$?
- Elipsoide:

$$\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2} + \frac{(z - z_0)^2}{c^2} = 1$$



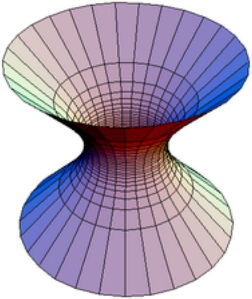
- $z =$const: elipses para $|z - z_0| \leq c$:

Superfíces Quadráticas

- Hiperboloide 1 folha:

$$\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}-\frac{(z-z_0)^2}{c^2}=1$$

- $z=$const: elipse, $x, y=$const: Hipérboles

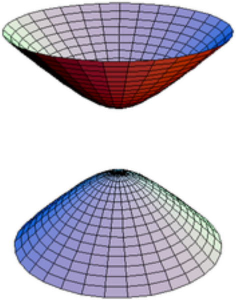


- Gerado por retas!

Superfíces Quadráticas

- Hiperboloide 2 folhas (z=const: elipse, x,y=c: hipérbole):

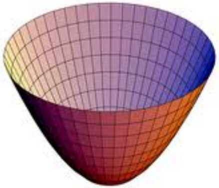
$$\frac{(x-x_0)^2}{a^2}-\frac{(y-y_0)^2}{b^2}-\frac{(z-z_0)^2}{c^2}=1$$



- Gerado por retas!

- Paraboloide Eliptica (z=const: elipse para $|z - z_0| \leq c$):

$$\frac{2(z - z_0)}{c} = \frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2}$$



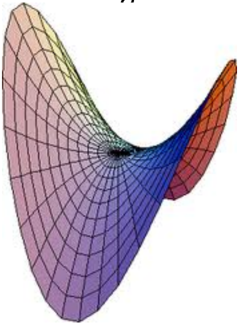
Superfíces Quadráticas

- Paraboloide Hiperbólica

$$\frac{2(z - z_0)}{c} = \frac{(x - x_0)^2}{a^2} - \frac{(y - y_0)^2}{b^2}$$

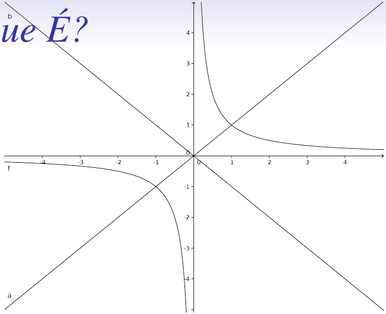
Ou:

$$\frac{2(z - z_0)}{c} = -\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2}$$



- Gerado por retas!

O Que Que É?



- $xy = 1$
- $y = f(x) = \frac{1}{x},\ x \in \mathbb{R} - \{0\}$

- Substituição Linear:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} =$$

$$\begin{pmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \frac{x'-y'}{\sqrt{2}} \\ \frac{x'+y'}{\sqrt{2}} \end{pmatrix}$$

- $1 = xy = \frac{x'-y'}{\sqrt{2}} \cdot \frac{x'+y'}{\sqrt{2}} = \frac{x'^2 - y'^2}{(\sqrt{2})^2}$

Formas Quadráticas

- Polinômios Homogêneos, ordem n: $P(\lambda x, \lambda y) = \lambda^n P(x, y)$
- No plano:

$$a_{xx}x^2+2a_{xy}xy+a_{yy}y^2 = (x\ y)\begin{pmatrix} a_{xx} & a_{xy} \\ a_{xy} & a_{yy} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \underline{\mathbf{r}}^T \underline{\underline{\mathbf{A}}}\,\underline{\mathbf{r}}$$

- No espaço:

$$a_{xx}x^2 + 2a_{xy}xy + 2a_{xz}xz + a_{yy}y^2 + a_{yz}yz + a_{zz}z^2 =$$

$$(x\ y\ z)\begin{pmatrix} a_{xx} & a_{xy} & a_{xz} \\ a_{xy} & a_{yy} & a_{yz} \\ a_{xz} & a_{yz} & a_{zz} \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underline{\mathbf{r}}^T \underline{\underline{\mathbf{A}}}\,\underline{\mathbf{r}}$$

- $\underline{\underline{\mathbf{A}}}$ simétricas: $\underline{\underline{\mathbf{A}}}^T = \underline{\underline{\mathbf{A}}}$

Sistema Linear Quadrático

- $(I): \underline{\underline{\mathbf{A}}}\, \underline{\mathbf{v}} = \underline{\mathbf{b}}$
- Se $\det \underline{\underline{\mathbf{A}}} \neq 0, \quad \exists! \underline{\underline{\mathbf{A}}}^{-1}: \quad \underline{\underline{\mathbf{A}}}\, \underline{\underline{\mathbf{A}}}^{-1} = \underline{\underline{\mathbf{A}}}^{-1} \underline{\underline{\mathbf{A}}} = \underline{\underline{\mathbf{I}}}$.
- Assim: $\underline{\mathbf{v}} = \underline{\underline{\mathbf{A}}}^{-1} \underline{\mathbf{b}}$.
- Matriz Total: $\underline{\underline{\mathbf{T}}} = \left(\underline{\underline{\mathbf{A}}} \mid \underline{\mathbf{b}} \right)$.
- Posto ρ: Ordem do maior subdeteterminante sendo $\neq 0$.
- $\rho \leq n$. Se $\det \underline{\underline{\mathbf{A}}} \neq 0$: $\rho = n$.
- (I) tem solução see: $\rho = \rho_{\underline{\underline{\mathbf{A}}}} = \rho_{\underline{\underline{\mathbf{T}}}}$.
- $S = \underline{\mathbf{v}}_0 + \sum_{i=1}^{n-\rho} t_i \underline{\mathbf{v}}_i$, $t_i \in \mathbb{R}$: SCSiH=SPSiH+SCSH.
- Uma $(n-\rho)$-infinidade de soluções: $\dim S = n - \rho$.

Aplicação Linear, $f : \mathbb{R}^n \to \mathbb{R}^n$

$$f(\underline{\mathbf{v}}) = \underline{\underline{\mathbf{A}}}\ \underline{\mathbf{v}}$$

- $\underline{\underline{\mathbf{A}}}$ matriz coluna:
- $\underline{\mathbf{a}}_i = f(\underline{\mathbf{e}}_i)$:

$$\underline{\underline{\mathbf{A}}} = \begin{pmatrix} | & | & & | \\ \underline{\mathbf{a}}_1 & \underline{\mathbf{a}}_2 & \cdots & \underline{\mathbf{a}}_n \\ | & | & & | \end{pmatrix}$$

$$f(\underline{\mathbf{e}}_i) = \begin{pmatrix} | & | & & | \\ \underline{\mathbf{a}}_1 & \underline{\mathbf{a}}_2 & \cdots & \underline{\mathbf{a}}_n \\ | & | & & | \end{pmatrix} \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix} \begin{matrix} \\ \\ \leftarrow i \\ \\ \\ \end{matrix} = \underline{\mathbf{a}}_i$$

- $Im(f) = ger(\underline{\mathbf{a}}_1, \ldots, \underline{\mathbf{a}}_n) = \{\underline{\mathbf{x}} \in \mathbb{R}^n |\ \underline{\mathbf{x}} = c_1\underline{\mathbf{a}}_1 + \ldots + c_n\underline{\mathbf{a}}_n\}$.
- $\dim Im(f) = \dim ger(\underline{\mathbf{a}}_1, \ldots, \underline{\mathbf{a}}_n) = \rho_{\underline{\underline{\mathbf{A}}}}$.

Autovetor/Autovalor

- Por quais $\lambda \in \mathbb{R}$ tem soluções *não-triviais*, $\underline{\mathbf{v}} \neq \underline{\mathbf{0}}$:

$$\underline{\underline{\mathbf{A}}}\ \underline{\mathbf{v}} = \lambda \underline{\mathbf{v}} = \lambda \underline{\underline{\mathbf{I}}}\ \underline{\mathbf{v}} \quad \Leftrightarrow \quad (\underline{\underline{\mathbf{A}}} - \lambda \underline{\underline{\mathbf{I}}})\underline{\mathbf{x}} = \underline{\mathbf{0}}$$

- Se $\underline{\underline{\mathbf{A}}}_\lambda = \underline{\underline{\mathbf{A}}} - \lambda \underline{\underline{\mathbf{I}}}$ regular:

$$\det \underline{\underline{\mathbf{A}}}_\lambda = \det (\underline{\underline{\mathbf{A}}} - \lambda \underline{\underline{\mathbf{I}}}) \neq 0,$$

  solução única: $\underline{\mathbf{v}} = \underline{\underline{\mathbf{A}}}_\lambda^{-1} \underline{\mathbf{0}} = \underline{\mathbf{0}}$. Trivialmente!!!!

- Soluções *não-triviais*, see λ raíz no *Polinômio Caraterístico*:

$$P_n(\lambda) = \det (\underline{\underline{\mathbf{A}}} - \lambda \underline{\underline{\mathbf{I}}}) = 0$$

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Polinômio Cararetístico I

$$P_n(\lambda) = \det(\underline{\underline{\mathbf{A}}} - \lambda \underline{\underline{\mathbf{I}}}) = \begin{vmatrix} a_{11}-\lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22}-\lambda & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn}-\lambda \end{vmatrix} =$$

$$(-1)^n \lambda^n - (-1)^n \lambda^{n-1} Tr(\underline{\underline{\mathbf{A}}}) + \cdots + \det \underline{\underline{\mathbf{A}}}$$

- Polinômio de grau n com coeficientes reais.
- *Máx.* n raízes reais.
- Número par de raízes complexas.
- Grau ímpar: Mín. uma raíz real.
- Raízes:

$$P_n(\lambda) = (\lambda_1 - \lambda) \cdots (\lambda_n - \lambda) =$$

$$P_n(\lambda) = (-1)^n \lambda^n - (-1)^n \lambda^{n-1} \sum_{i=1}^{n} \lambda_i + \cdots + \prod_{i=1}^{n} \lambda_i$$

Polinômio Cararetístico II

$$P(\lambda) = (-1)^n \lambda^n - (-1)^n \lambda^{n-1} Tr(\underline{\underline{\mathbf{A}}}) + \cdots + \det \underline{\underline{\mathbf{A}}}$$

$$P(\lambda) = (-1)^n \lambda^n - (-1)^n \lambda^{n-1} \sum_{i=1}^{n} \lambda_i + \cdots + \prod_{i=1}^{n} \lambda_i$$

- $\sum_{i=1}^{n} \lambda_i = Tr(\underline{\underline{\mathbf{A}}}) = a_{11} + \ldots + a_{nn}$.
- $\prod_{i=1}^{n} \lambda_i = \det \underline{\underline{\mathbf{A}}}$.

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Auto-espaços I

$$S_\lambda = \{\underline{\mathbf{v}} \in \mathbb{R}^n \mid \underline{\underline{\mathbf{A}}}\ \underline{\mathbf{v}} = \lambda \underline{\mathbf{v}}\}$$

- Espaçõs vetoriais:

$$(i) : \underline{\mathbf{v}} \in S_\lambda, c \in \mathbb{R} \Rightarrow c\underline{\mathbf{v}} \in S_\lambda$$

$$(ii) : \underline{\mathbf{v}}, \underline{\mathbf{w}} \in S_\lambda \Rightarrow \underline{\mathbf{v}} + \underline{\mathbf{w}} \in S_\lambda$$

- $P(\lambda) \neq 0$: $S_\lambda = \{\underline{\mathbf{0}}\}$.
- $P(\lambda) = 0$: $1 \leq \dim S_\lambda \leq n$.
- $\bigcup_{\lambda_i \in \mathbb{R}} S_{\lambda_i} \subset \mathbb{R}^n$.

Auto-espaços II

- *Teorema*: Autovetores de autovalores diferentes s̃ linearmente independentes (LI).
- Dado $\lambda_1 \neq \lambda_2$, e $\underline{\mathbf{v}}_1, \underline{\mathbf{v}}_1 \neq \underline{\mathbf{0}}$:

$$\underline{\underline{\mathbf{A}}}\ \underline{\mathbf{v}}_1 = \lambda_1 \underline{\mathbf{v}}_1 \quad \wedge \quad \underline{\underline{\mathbf{A}}}\ \underline{\mathbf{v}}_2 = \lambda_2 \underline{\mathbf{v}}_2$$

- Suponhe $\underline{\mathbf{v}}_1, \underline{\mathbf{v}}_2$ linearmente dependentes (LD):

$$\underline{\mathbf{v}}_1 = c\underline{\mathbf{v}}_2 \Rightarrow \underline{\underline{\mathbf{A}}} c\underline{\mathbf{v}}_1 = \lambda_2 c\underline{\mathbf{v}}_1 \Leftrightarrow$$

$$\underline{\underline{\mathbf{A}}}\underline{\mathbf{v}}_1 = \lambda_2 \underline{\mathbf{v}}_1 \Leftrightarrow \lambda_1 \underline{\mathbf{v}}_1 = \lambda_2 \underline{\mathbf{v}}_1 \Leftrightarrow$$

$$(\lambda_1 - \lambda_2)\underline{\mathbf{v}}_1 = \underline{\mathbf{0}} \Leftrightarrow \lambda_1 = \lambda_2 \vee \underline{\mathbf{v}}_1 = \underline{\mathbf{0}}$$

  Absurdo!

- $\lambda_1 \neq \lambda_2$: $S_{\lambda_1} \bigcap S_{\lambda_2} = \{\underline{\mathbf{0}}\}$

Auto-espaços III

- 'Perder AVs':
  - $\lambda_i \in \mathbb{C} - \mathbb{R}$: $\overline{\lambda_i} \neq \lambda_i$;
  - $dim S_{\lambda_i} < \rho_{\lambda_i}$.
- $dim \bigcup_{\lambda_i \in \mathbb{R}} S_{\lambda_i} = n \quad \Rightarrow \quad \exists$ base de autovetores!
- <u>Se</u> $\exists$ base de autovetores: $\underline{\mathbf{v}}_1, \ldots, \underline{\mathbf{v}}_n$:

$$\underline{\underline{\mathbf{A}}}\ \underline{\mathbf{v}}_i = \lambda_i \underline{\mathbf{v}}_i = [0, \ldots, \lambda_i, \ldots, 0]^T_{\underline{\mathbf{v}}}$$

Matrizes Simétricas

- *Teorema*: Se $\underline{\underline{\mathbf{A}}}$ é *simétrica*, todos os autovalores s̃ reais.
- Meu Pai criou esse Matemática meeeeesmo....
- Lembre-se: $(\underline{\underline{\mathbf{A}}}\,\underline{\underline{\mathbf{B}}})^T = \underline{\underline{\mathbf{B}}}^T\underline{\underline{\mathbf{A}}}^T$, $\overline{\underline{\underline{\mathbf{A}}}\,\underline{\underline{\mathbf{B}}}} = \overline{\underline{\underline{\mathbf{A}}}}\,\overline{\underline{\underline{\mathbf{B}}}}$.
- E em $\mathbb{C}$: $\underline{\mathbf{v}} \cdot \underline{\mathbf{w}} = \overline{\underline{\mathbf{v}}}^T\underline{\mathbf{w}}$, (garantindo: $|\underline{\mathbf{v}}|^2 \geq 0$).
- $\rightarrow$: $\lambda \in \mathbb{R} \Rightarrow \underline{\mathbf{v}} \in \mathbb{R}^n$!
- Suponhe: $\lambda \notin \mathbb{R}$ $(\overline{\lambda} \neq \lambda)$ e $\underline{\mathbf{v}} \neq \underline{\mathbf{0}}$:

$$\underline{\underline{\mathbf{A}}}\,\underline{\mathbf{v}} = \lambda\underline{\mathbf{v}} \quad \Rightarrow \quad \overline{\underline{\mathbf{v}}}^T\underline{\underline{\mathbf{A}}}\,\underline{\mathbf{v}} = \lambda|\underline{\mathbf{v}}|^2 \in \mathbb{C} \quad \Rightarrow \quad (\lambda|\underline{\mathbf{v}}|^2)^T = \lambda|\underline{\mathbf{v}}|^2$$

- $\overline{\lambda}|\underline{\mathbf{v}}|^2 = (\overline{\lambda}|\underline{\mathbf{v}}|^2)^T = (\overline{\lambda|\underline{\mathbf{v}}|^2})^T = (\overline{\overline{\underline{\mathbf{v}}}^T\underline{\underline{\mathbf{A}}}\,\underline{\mathbf{v}}})^T = (\underline{\mathbf{v}}^T\overline{\underline{\underline{\mathbf{A}}}}\,\overline{\underline{\mathbf{v}}})^T =$
$(\underline{\mathbf{v}}^T\underline{\underline{\mathbf{A}}}\,\overline{\underline{\mathbf{v}}})^T = \overline{\underline{\mathbf{v}}}^T\underline{\underline{\mathbf{A}}}\,\underline{\mathbf{v}} = \lambda|\underline{\mathbf{v}}|^2 \Leftrightarrow (\lambda - \overline{\lambda})|\underline{\mathbf{v}}|^2 = 0 \Leftrightarrow$

$$\lambda = \overline{\lambda} \Leftrightarrow \lambda \in \mathbb{R} \quad \square$$

Matrizes Simétricas: $\underline{\underline{\mathbf{A}}} = \underline{\underline{\mathbf{A}}}^T$

- *Teorema*: Autovetores de autovalores diferentes, s̃ ortogonais.
- Meu Pai éeeeeeeeeee divino!
- Suponhe: $\underline{\underline{\mathbf{A}}}\,\underline{\mathbf{v}}_1 = \lambda_1 \underline{\mathbf{v}}_1$ e $\underline{\underline{\mathbf{A}}}\,\underline{\mathbf{v}}_2 = \lambda_2 \underline{\mathbf{v}}_2$ com $\lambda_1 \neq \lambda_2$.
- $\underline{\mathbf{v}}_2^T \underline{\underline{\mathbf{A}}}\,\underline{\mathbf{v}}_1 = \underline{\mathbf{v}}_2^T \lambda_1 \underline{\mathbf{v}}_1 = \lambda_1 \underline{\mathbf{v}}_1 \cdot \underline{\mathbf{v}}_2$.
- $\underline{\mathbf{v}}_1^T \underline{\underline{\mathbf{A}}}\,\underline{\mathbf{v}}_2 = \underline{\mathbf{v}}_1^T \lambda_2 \underline{\mathbf{v}}_2 = \lambda_2 \underline{\mathbf{v}}_1 \cdot \underline{\mathbf{v}}_2$.
- Por ser um *número*: $\underline{\mathbf{v}}_1^T \underline{\underline{\mathbf{A}}}\,\underline{\mathbf{v}}_2 = (\underline{\mathbf{v}}_1^T \underline{\underline{\mathbf{A}}}\,\underline{\mathbf{v}}_2)^T = \underline{\mathbf{v}}_2^T \underline{\underline{\mathbf{A}}}\,\underline{\mathbf{v}}_1$
- Conclusão: $\lambda_1 \underline{\mathbf{v}}_1 \cdot \underline{\mathbf{v}}_2 = \lambda_2 \underline{\mathbf{v}}_1 \cdot \underline{\mathbf{v}}_2 \Leftrightarrow (\lambda_1 - \lambda_2) \underline{\mathbf{v}}_1 \cdot \underline{\mathbf{v}}_2 = 0 \Leftrightarrow \underline{\mathbf{v}}_1 \cdot \underline{\mathbf{v}}_2 = 0$
- *Teorema*: Se λ_i raíz de ordem ρ_i em $P_n(\lambda)$: dim $S_{\lambda_i} = \rho_i$.
- Podemos ortogonalizar espaços (Gram-Scmidt)!

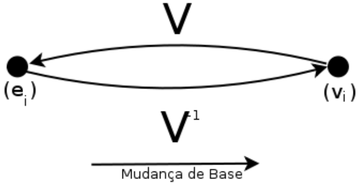
Mudança de Base I

- Base novo: $(\underline{\mathbf{v}}_1, \ldots, \underline{\mathbf{v}}_n)$, formamos:

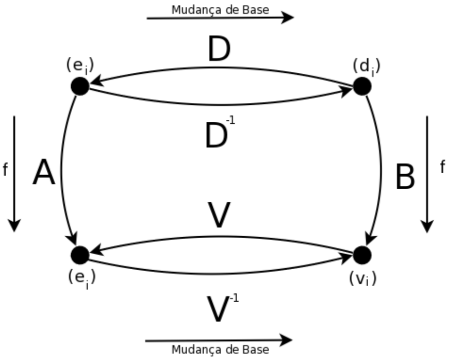
$$\underline{\underline{\mathbf{V}}} = \begin{pmatrix} | & & | \\ \underline{\mathbf{v}}_1 & \cdots & \underline{\mathbf{v}}_n \\ | & & | \end{pmatrix}$$

- Coordenados antigos, $\underline{\mathbf{v}}$, e novos, $\underline{\mathbf{v}}'$.

$$\underline{\mathbf{v}} = \underline{\underline{\mathbf{V}}}\,\underline{\mathbf{v}}' \qquad \Leftrightarrow \qquad \underline{\mathbf{v}}' = \underline{\underline{\mathbf{V}}}^{-1}\underline{\mathbf{v}}$$



Mudança de Base II



- $(\underline{\underline{\mathbf{D}}} = \underline{\underline{\mathbf{V}}})$
- $\underline{\underline{\mathbf{B}}} = \underline{\underline{\mathbf{V}}}^{-1}\underline{\underline{\mathbf{A}}}\,\underline{\underline{\mathbf{V}}}$.
- $\underline{\underline{\mathbf{A}}}$ e $\underline{\underline{\mathbf{B}}}$ *similares*: $\underline{\underline{\mathbf{A}}} \sim \underline{\underline{\mathbf{B}}} \Leftrightarrow \exists \underline{\underline{\mathbf{V}}} : \underline{\underline{\mathbf{B}}} = \underline{\underline{\mathbf{V}}}^{-1}\underline{\underline{\mathbf{A}}}\,\underline{\underline{\mathbf{V}}}$.
- Mesma aplicação linear/forma quadrática/curva em bases diferentes.

Matrizes Similares I

- Similaridade ($\sim$) Relação de Equivalência:
- Reflexiva: (sou meu próprio amigo):

$$\underline{\underline{\mathbf{A}}} = \underline{\mathbf{I}}^{-1}\underline{\underline{\mathbf{A}}}\,\underline{\mathbf{I}}$$

- Simétrica (sou amigo do meus amigo e vice-versa).

$$\underline{\underline{\mathbf{B}}} = \underline{\underline{\mathbf{V}}}^{-1}\underline{\underline{\mathbf{A}}}\,\underline{\underline{\mathbf{V}}} \Leftrightarrow \underline{\underline{\mathbf{A}}} = \underline{\underline{\mathbf{V}}}\,\underline{\underline{\mathbf{A}}}\,\underline{\underline{\mathbf{V}}}^{-1}$$

- Transitiva (sou amigo dos amigos dos meus amigos).

$$\underline{\underline{\mathbf{B}}} = \underline{\underline{\mathbf{V}}}^{-1}\underline{\underline{\mathbf{A}}}\,\underline{\underline{\mathbf{V}}} \wedge \underline{\underline{\mathbf{C}}} = \underline{\underline{\mathbf{W}}}^{-1}\underline{\underline{\mathbf{B}}}\,\underline{\underline{\mathbf{W}}} \Rightarrow$$

$$\underline{\underline{\mathbf{C}}} = \underline{\underline{\mathbf{W}}}^{-1}\underline{\underline{\mathbf{B}}}\,\underline{\underline{\mathbf{W}}} = \underline{\underline{\mathbf{W}}}^{-1}(\underline{\underline{\mathbf{V}}}^{-1}\underline{\underline{\mathbf{A}}}\,\underline{\underline{\mathbf{V}}})\underline{\underline{\mathbf{W}}} =$$

$$(\underline{\underline{\mathbf{W}}}^{-1}\underline{\underline{\mathbf{V}}}^{-1})\underline{\underline{\mathbf{A}}}(\underline{\underline{\mathbf{VW}}}) = (\underline{\underline{\mathbf{VW}}})^{-1}\underline{\underline{\mathbf{A}}}(\underline{\underline{\mathbf{VW}}})$$

Matrizes Similares II

- Matrizes Similares tem mesmo Determinante:

$$\det \underline{\underline{\mathbf{B}}} = \det\left(\underline{\underline{\mathbf{V}}}^{-1}\underline{\underline{\mathbf{A}}}\;\underline{\underline{\mathbf{V}}}\right) = \det\left(\underline{\underline{\mathbf{V}}}^{-1}\right)\det \underline{\underline{\mathbf{A}}}\det \underline{\underline{\mathbf{V}}} =$$

$$\left(\det \underline{\underline{\mathbf{V}}}\right)^{-1} det\underline{\underline{\mathbf{A}}}\det \underline{\underline{\mathbf{V}}} = det\underline{\underline{\mathbf{A}}}$$

- Matrizes Similares tem mesmo Polinômio Caraterístico:

$$\det\left(\underline{\underline{\mathbf{B}}} - \lambda\underline{\underline{\mathbf{I}}}\right) = \det\left(\underline{\underline{\mathbf{V}}}^{-1}\underline{\underline{\mathbf{A}}}\;\underline{\underline{\mathbf{V}}} - \lambda\underline{\underline{\mathbf{I}}}\right) =$$

$$\det\left(\underline{\underline{\mathbf{V}}}^{-1}\underline{\underline{\mathbf{A}}}\;\underline{\underline{\mathbf{V}}} - \lambda\underline{\underline{\mathbf{V}}}^{-1}\underline{\underline{\mathbf{I}}}\;\underline{\underline{\mathbf{V}}}\right) = \det\left(\underline{\underline{\mathbf{V}}}^{-1}\left(\underline{\underline{\mathbf{A}}} - \lambda\underline{\underline{\mathbf{I}}}\right)\underline{\underline{\mathbf{V}}}\right) =$$

$$\det\left(\underline{\underline{\mathbf{A}}} - \lambda\underline{\underline{\mathbf{I}}}\right)$$

Matrizes Similares III

- Matrizes Similares tem mesmo Autovalores.
- Os autovalores š *Invariantes* de uma mudança de Base.
- Os coeficientes do Polinômio Caraterístico š *Invariantes*.
- Matrizes Similares tem mesmo Traço: $Tr\underline{\underline{\mathbf{A}}} = \sum_i \lambda_i$.
- Matrizes Similares š... Similares.

Base de Autovetores

- Em base $\underline{\mathbf{v}}_1, \ldots, \underline{\mathbf{v}}_n$:

$$\lambda_i \underline{\mathbf{v}}_i = [0, \cdots, \overset{\overset{i}{\downarrow}}{\lambda_i}, \cdots, 0]^T_{\underline{\mathbf{v}}}$$

- $\underline{\underline{\mathbf{B}}}$ diagonal:

$$\underline{\underline{\mathbf{B}}} = \begin{pmatrix} | & | & & | \\ f(\underline{\mathbf{v}}_1) & f(\underline{\mathbf{v}}_2) & \cdots & f(\underline{\mathbf{v}}_n) \\ | & | & & | \end{pmatrix} =$$

$$\begin{pmatrix} | & | & & | \\ \lambda_1 \underline{\mathbf{v}}_1 & \lambda_2 \underline{\mathbf{v}}_2 & \cdots & \lambda_n \underline{\mathbf{v}}_n \\ | & | & & | \end{pmatrix} = \begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{pmatrix}$$

- *Teorema*: Se $\underline{\underline{\mathbf{A}}} = \underline{\underline{\mathbf{A}}}^T$, existe um base, $\underline{\mathbf{v}}_1, \ldots, \underline{\mathbf{v}}_n$, *ortonormal*: $\underline{\mathbf{v}}_i \cdot \underline{\mathbf{v}}_j = \delta_{ij}$ de autovetores: $\underline{\underline{\mathbf{A}}}\, \underline{\mathbf{v}}_i = \lambda_i \underline{\mathbf{v}}_i$.

Matrizes Ortogonais

Base Ortonormal em Colunas:

- $$\underline{\underline{V}} = \begin{pmatrix} | & | & & | \\ \underline{v}_1 & \underline{v}_2 & \cdots & \underline{v}_n \\ | & | & & | \end{pmatrix} \quad \Leftrightarrow \quad \underline{\underline{V}}^T = \begin{pmatrix} - & \underline{v}_1 & - \\ - & \underline{v}_2 & - \\ & \vdots & \\ - & \underline{v}_n & - \end{pmatrix}$$
- $$\underline{\underline{V}}^T\underline{\underline{V}} = \begin{pmatrix} - & \underline{v}_1 & - \\ - & \underline{v}_2 & - \\ & \vdots & \\ - & \underline{v}_n & - \end{pmatrix} \begin{pmatrix} | & | & & | \\ \underline{v}_1 & \underline{v}_2 & \cdots & \underline{v}_n \\ | & | & & | \end{pmatrix}$$
  $$\begin{pmatrix} \underline{v}_1 \cdot \underline{v}_1 & \underline{v}_1 \cdot \underline{v}_2 & \cdots & \underline{v}_1 \cdot \underline{v}_n \\ \underline{v}_2 \cdot \underline{v}_1 & \underline{v}_2 \cdot \underline{v}_2 & \cdots & \underline{v}_2 \cdot \underline{v}_n \\ \vdots & \vdots & \ddots & \vdots \\ \underline{v}_n \cdot \underline{v}_1 & \underline{v}_n \cdot \underline{v}_2 & \cdots & \underline{v}_n \cdot \underline{v}_n \end{pmatrix} = (\underline{v}_i \cdot \underline{v}_j) = (\delta_{ij}) = \underline{\underline{I}} \Leftrightarrow$$
  $$\underline{\underline{V}}^{-1} = \underline{\underline{V}}^T$$
- $\left(\det \underline{\underline{V}}\right)^2 = \det \underline{\underline{I}} = 1 \Leftrightarrow \det \underline{\underline{V}} = \pm 1.$

Caso $\mathbb{C}$omplexo

- $\underline{\underline{\mathbf{A}}}$ Hermitiano: $\underline{\underline{\mathbf{A}}}^T = \overline{\underline{\underline{\mathbf{A}}}}$.
- Hermitiano $\Rightarrow$ autovalores reais.
- Hermitiano $\Rightarrow$ autovetores ortogonalizáveis.
- $\underline{\underline{\mathbf{U}}}$ Unitário: $\underline{\underline{\mathbf{U}}}^{-1} = \overline{\underline{\underline{\mathbf{U}}}}^T$.

Base de Autovetores Ortonormais

- $\underline{\underline{\mathbf{B}}} = \underline{\underline{\mathbf{V}}}^{-1}\underline{\underline{\mathbf{A}}}\;\underline{\underline{\mathbf{V}}} = \underline{\underline{\mathbf{V}}}^{T}\underline{\underline{\mathbf{A}}}\;\underline{\underline{\mathbf{V}}} =$

$$\begin{pmatrix} - & \underline{\mathbf{v}}_1 & - \\ - & \underline{\mathbf{v}}_2 & - \\ & \vdots & \\ - & \underline{\mathbf{v}}_n & - \end{pmatrix} \begin{pmatrix} | & | & & | \\ \lambda_1\underline{\mathbf{v}}_1 & \lambda_2\underline{\mathbf{v}}_2 & \cdots & \lambda_n\underline{\mathbf{v}}_n \\ | & | & & | \end{pmatrix} =$$

$$\begin{pmatrix} - & \underline{\mathbf{v}}_1 & - \\ - & \underline{\mathbf{v}}_2 & - \\ & \vdots & \\ - & \underline{\mathbf{v}}_n & - \end{pmatrix} \underline{\underline{\mathbf{A}}} \begin{pmatrix} | & | & & | \\ \underline{\mathbf{v}}_1 & \underline{\mathbf{v}}_2 & \cdots & \underline{\mathbf{v}}_n \\ | & | & & | \end{pmatrix} =$$

$$\begin{pmatrix} \lambda_1\underline{\mathbf{v}}_1\cdot\underline{\mathbf{v}}_1 & \lambda_2\underline{\mathbf{v}}_1\cdot\underline{\mathbf{v}}_2 & \cdots & \lambda_n\underline{\mathbf{v}}_1\cdot\underline{\mathbf{v}}_n \\ \lambda_1\underline{\mathbf{v}}_2\cdot\underline{\mathbf{v}}_1 & \lambda_2\underline{\mathbf{v}}_2\cdot\underline{\mathbf{v}}_2 & \cdots & \lambda_n\underline{\mathbf{v}}_2\cdot\underline{\mathbf{v}}_n \\ \vdots & \vdots & & \vdots \\ \lambda_1\underline{\mathbf{v}}_n\cdot\underline{\mathbf{v}}_1 & \lambda_2\underline{\mathbf{v}}_n\cdot\underline{\mathbf{v}}_2 & \cdots & \lambda_n\underline{\mathbf{v}}_n\cdot\underline{\mathbf{v}}_n \end{pmatrix} = (\lambda_i\delta_{ij})$$

Base de Autovetores Ortonormais

- $F(x,y) = \underline{\mathbf{r}}^T \underline{\underline{\mathbf{A}}}\ \underline{\mathbf{r}} = \underline{\mathbf{r}}'^T \underline{\underline{\mathbf{B}}}\ \underline{\mathbf{r}}' = \lambda_1 x'^2 + \lambda_2 y'^2$.
  $F(x,y,z) = \lambda_1 x'^2 + \lambda_2 y'^2 + \lambda_3 z'^2$.
- Termos 'mistos' Eliminados!!!!

Problemas resolvemos na hora.
As Milagres demoram mais um pouco...

- $\underline{\mathbf{r}} = \underline{\underline{\mathbf{V}}}\ \underline{\mathbf{r}}' \Leftrightarrow \underline{\mathbf{r}}' = \underline{\underline{\mathbf{V}}}^T \underline{\mathbf{r}}$.
- $j =$

$$\overbrace{ax^2 + by^2 + cz^2 + 2dxy + 2exz + 2fyz}^{Q} + \overbrace{gx + hy + iz}^{L} =$$

$$\underline{\mathbf{r}}^T \underline{\underline{\mathbf{A}}}\ \underline{\mathbf{r}} + \underline{\mathbf{b}}^T \underline{\mathbf{r}} = (\underline{\underline{\mathbf{V}}}\ \underline{\mathbf{r}}')^T \underline{\underline{\mathbf{A}}}\ (\underline{\underline{\mathbf{V}}}\ \underline{\mathbf{r}}') + \underline{\mathbf{b}}^T (\underline{\underline{\mathbf{V}}}\ \underline{\mathbf{r}}') =$$

$$\underline{\mathbf{r}}'^T (\underline{\underline{\mathbf{V}}}^T \underline{\underline{\mathbf{A}}}\ \underline{\underline{\mathbf{V}}}) \underline{\mathbf{r}} + (\underline{\underline{\mathbf{V}}}^T \underline{\mathbf{b}})^T \underline{\mathbf{r}}' = \underline{\mathbf{r}}'^T \underline{\underline{\mathbf{B}}} \underline{\mathbf{r}} + \underline{\mathbf{b}}'^T \underline{\mathbf{r}}'$$

Caso $n = 2$*: Curvas Quadráticas*

- $\underline{\underline{\mathbf{A}}} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$
-
$$\det(\underline{\underline{\mathbf{A}}} - \lambda \underline{\underline{\mathbf{I}}}) = \begin{vmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{vmatrix} = (a_{11} - \lambda)(a_{22} - \lambda) - a_{12}a_{21} =$$
$$\lambda^2 - (a_{11} + a_{22})\lambda + a_{11}a_{22} - a_{12}a_{21} = \lambda^2 - Tr\underline{\underline{\mathbf{A}}}\lambda + \det \underline{\underline{\mathbf{A}}}$$
- $\Delta = (a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{12}a_{21}) = (a_{11} - a_{22})^2 + 4a_{12}a_{21}$
- $\underline{\underline{\mathbf{A}}} = \underline{\underline{\mathbf{A}}}^T$:
  - $\Delta = (a_{11} - a_{22})^2 + 4a_{12}^2 \geq 0$;
  - $\Delta = 0 \Leftrightarrow a_{11} = a_{22} := a \wedge a_{12} = 0 \Leftrightarrow \underline{\underline{\mathbf{A}}} = a\,\underline{\underline{\mathbf{I}}}$: Já diagonal!
-
$$\lambda_{\pm} = \frac{a_{11} + a_{22} \pm \sqrt{\Delta}}{2}$$

$$\lambda = \lambda_\pm = \frac{a_{11}+a_{22}\pm\sqrt{\Delta}}{2}$$

$$\underline{\underline{\mathbf{A}}} - \lambda_\pm \underline{\underline{\mathbf{I}}} = \begin{pmatrix} a_{11}-\lambda_\pm & a_{12} \\ a_{21} & a_{22}-\lambda_\pm \end{pmatrix} =$$

$$\begin{pmatrix} a_{11} - \frac{a_{11}+a_{22}\mp\sqrt{\Delta}}{2} & a_{12} \\ a_{21} & a_{22} - \frac{a_{11}+a_{22}\mp\sqrt{\Delta}}{2} \end{pmatrix} \sim$$

$$\begin{pmatrix} a_{11}-a_{22}\mp\sqrt{\Delta} & 2a_{12} \\ 2a_{21} & a_{22}-a_{11}\mp\sqrt{\Delta} \end{pmatrix} \sim$$

$$\begin{pmatrix} a_{11}-a_{22}\mp\sqrt{\Delta} & 2a_{12} \\ 0 & 0 \end{pmatrix} \underline{\mathbf{x}} = \underline{\mathbf{0}} \Leftrightarrow$$

$$x_1(a_{11}-a_{22}\mp\sqrt{\Delta}) + 2x_2 2a_{12} = 0$$

$\lambda = \lambda_\pm = \frac{a_{11}+a_{22}\pm\sqrt{\Delta}}{2}$, *cont.:*

- Solução completa; $t, t' \in \mathbb{R}$:

$$\underline{\mathbf{x}}_\pm = t \underbrace{\begin{pmatrix} -2a_{12} \\ a_{11} - a_{22} \mp \sqrt{\Delta} \end{pmatrix}}_{\underline{\mathbf{v}}_\pm} = t' \underbrace{\begin{pmatrix} a_{22} - a_{11} \mp \sqrt{\Delta} \\ -2a_{21} \end{pmatrix}}_{\underline{\mathbf{w}}_\pm}$$

- $\underline{\mathbf{v}}_+ \parallel \underline{\mathbf{w}}_+ \wedge \underline{\mathbf{v}}_- \parallel \underline{\mathbf{w}}_-$.
- $\underline{\mathbf{v}}_+ \cdot \underline{\mathbf{v}}_- = 4a_{12}^2 + (a_{11} - a_{22} + \sqrt{\Delta})(a_{11} - a_{22} - \sqrt{\Delta})$

$$4a_{12}^2+(a_{11}-a_{22})^2-\Delta = 4a_{12}^2+(a_{11}-a_{22})^2-((a_{11}-a_{22})^2+4a_{12}a_{21})$$

$$= 4a_{12}(a_{12} - a_{21}) = -\underline{\mathbf{v}}_- \cdot \underline{\mathbf{w}}_+$$

- Se $a_{12} = a_{21}$: $\underline{\mathbf{v}}_+ \perp \underline{\mathbf{v}}_- \wedge \underline{\mathbf{w}}_+ \perp \underline{\mathbf{w}}_-$.
- $\widehat{\underline{\mathbf{v}}}_\pm = \underline{\mathbf{w}}_\mp$.

$\lambda = \lambda_\pm = \frac{a_{11}+a_{22}\pm\sqrt{\Delta}}{2}$, *cont.:*

- Escolhemos:

$$\underline{\mathbf{v}}_1 = \underline{\mathbf{v}}_+ = \begin{pmatrix} -2a_{12} \\ a_{11} - a_{22} \mp \sqrt{\Delta} \end{pmatrix}, \qquad \underline{\widetilde{\mathbf{v}}}_1 = \frac{\underline{\mathbf{v}}_1}{|\underline{\mathbf{v}}_1|}$$

$$\underline{\mathbf{v}}_2 = \underline{\widehat{\mathbf{v}}}_+ = \underline{\mathbf{w}}_- = \begin{pmatrix} a_{22} - a_{11} \mp \sqrt{\Delta} \\ -2a_{21} \end{pmatrix}, \qquad \underline{\widetilde{\mathbf{v}}}_2 = \frac{\underline{\mathbf{v}}_2}{|\underline{\mathbf{v}}_2|}$$

- Substituição ortogonal:

$$\underline{\underline{\mathbf{V}}} = \begin{pmatrix} | & | \\ \underline{\widetilde{\mathbf{v}}}_1 & \underline{\widetilde{\mathbf{v}}}_2 \\ | & | \end{pmatrix}$$

- $\det \underline{\underline{\mathbf{V}}} = +1 \Rightarrow (\underline{\widetilde{\mathbf{v}}}_1, \underline{\widetilde{\mathbf{v}}}_2)$ orientados *positivamente*.

$2xy = 1$:

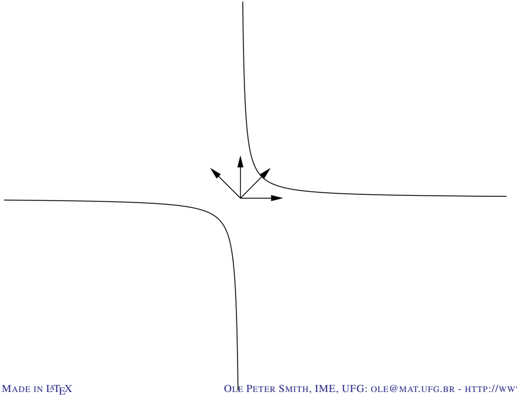
- $$2xy = \begin{pmatrix} x & y \end{pmatrix}^T \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$
- $$f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ x \end{pmatrix}$$
- Imediato, autovetores: $\underline{\mathbf{v}}_1 = (1, 1)^T$ e $\underline{\mathbf{v}}_2 = (-1, 1)^T$!
- Autovalores 1 resp. -1!
- $$\det\left(\underline{\underline{\mathbf{A}}} - \underline{\underline{\mathbf{I}}}\right) = \begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 - 1 = 0 \Leftrightarrow \lambda = \pm 1$$

$2xy = 1$, *cont.:*

- $\lambda_1 = 1$:
$$\underline{\underline{\mathbf{A}}} - \underline{\underline{\mathbf{I}}} = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \sim \begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix} \underline{\mathbf{x}} = \underline{\mathbf{0}} \Leftrightarrow \underline{\mathbf{x}} = t \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \ t \in \mathbb{R}$$
$$\underline{\mathbf{v}}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \Rightarrow \quad \underline{\widetilde{\mathbf{v}}}_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$
- $\lambda_2 = -1$:
$$\underline{\underline{\mathbf{A}}} + \underline{\underline{\mathbf{I}}} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \underline{\mathbf{x}} = \underline{\mathbf{0}} \Leftrightarrow \underline{\mathbf{x}} = t \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \ t \in \mathbb{R}$$
$$\underline{\mathbf{v}}_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad \Rightarrow \quad \underline{\widetilde{\mathbf{v}}}_2 = \widehat{\underline{\widetilde{\mathbf{v}}}}_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$
- Subst. ortogonal:
$$\underline{\underline{\mathbf{V}}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{pmatrix};$$
- $\det \underline{\underline{\mathbf{V}}} = 1$

$2xy = 1$, *cont.:*

- $$\underline{\underline{\mathbf{B}}} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \underline{\underline{\mathbf{V}}}^{-1}\underline{\underline{\mathbf{A}}}\ \underline{\underline{\mathbf{V}}} = \underline{\underline{\mathbf{V}}}^{T}\underline{\underline{\mathbf{A}}}\ \underline{\underline{\mathbf{V}}}$$
- $1 = 2xy = x'^2 - y'^2$: Hiperbole, $C(0,0)$ e $a = b = 1$.



$2xy - \sqrt{2}ax = c$:

- $$\underline{\mathbf{b}} = -\begin{pmatrix} \sqrt{2}a \\ 0 \end{pmatrix}$$
- $$\underline{\mathbf{b}}' = -\underline{\underline{\mathbf{V}}}^T \begin{pmatrix} \sqrt{2}a \\ 0 \end{pmatrix} = -\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{2}a \\ 0 \end{pmatrix} = \begin{pmatrix} -a \\ a \end{pmatrix}$$
- $\underline{\mathbf{b}}^T \underline{\mathbf{r}} = \underline{\mathbf{b}}'^T \underline{\mathbf{r}}' = -ax' + ay'$.
- $c = 2xy - \sqrt{2}a = x'^2 - y'^2 - ax' + ay' =$
  $(x'^2 - ax') - (y'^2 - ay') = \left(x' - \frac{a}{2}\right)^2 - \left(\frac{a}{2}\right)^2 - \left(y' - \frac{a}{2}\right)^2 - \left(\frac{a}{2}\right)^2$
  $$\left(x' - \frac{a}{2}\right)^2 - \left(y' - \frac{a}{2}\right)^2 - \frac{a^2}{2} \qquad \Leftrightarrow$$
  $$\left(x' - \frac{a}{2}\right)^2 - \left(y' - \frac{a}{2}\right)^2 = c + \frac{a^2}{2}$$

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$2xy - \sqrt{2}ax = c$, *cont.:*

- Hiperbole, roteado ângulo $\frac{\pi}{4}$.
- Centro: $\underline{\mathbf{c}}' = \left(\frac{a}{2}, \frac{a}{2}\right)^T$.

$$\underline{\mathbf{c}} = \underline{\underline{\mathbf{V}}}\,\underline{\mathbf{c}}' = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} \frac{a}{2} \\ \frac{a}{2} \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{a}{\sqrt{2}} \end{pmatrix}$$

- Semi-eixos: $a = b = \sqrt{|c + \frac{a^2}{2}|}$.
- Assintodas: $y' - \frac{a}{2} = \pm(x' - \frac{a}{2}) \Leftrightarrow x = 0 \vee y = \frac{a}{\sqrt{2}}$.
- $c + \frac{a^2}{2} > 0$: 1^0 e 3^0 quadrante.
- $c + \frac{a^2}{2} = 0$: As assintodas.
- $c + \frac{a^2}{2} < 0$: 2^0 e 4^0 quadrante.

Estes são meus Princípios.
Se Você não Gosta deles, eu tenho Outros...
Groucho Marx

Life *sure* is a Mystery to be Lived
Not a Problem to be Solved
Please Always Enjoy!

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